M2 JUNE 03

1) $v = 6t - 2t^2$ $\quad v = 0 \Rightarrow 6\cancel{t} = 2t^{\cancel{2}}$ $\quad t = 3\text{sec}.$

$S = \int v\,dt = 3t^2 - \frac{2}{3}t^3 + C$

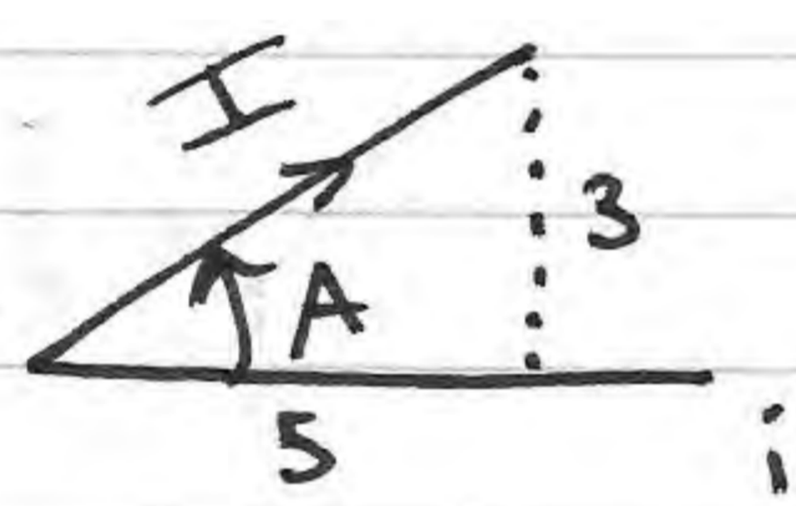
$t=0, S=0 \Rightarrow C=0 \qquad S = 3(3)^2 - \frac{2}{3}(3)^3 = \underline{9m}$

2) Momentum before $= 0.2(-10i) = -2i$
Momentum after $= 0.2(15i + 15j) = 3i + 3j$

Impulse = Change in Momentum $= 5i + 3j$

Magnitude $= \sqrt{5^2+3^2} = \underline{\sqrt{34}\text{ Ns}}$

b)

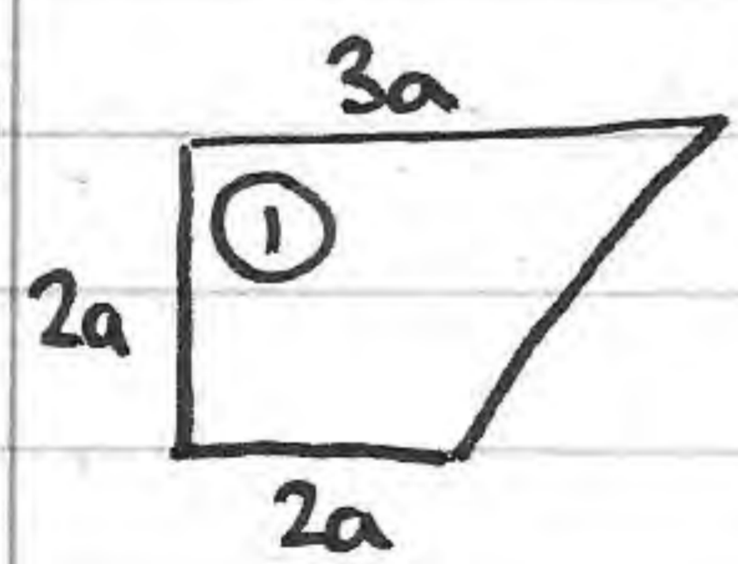


$A = \tan^{-1}\left(\frac{3}{5}\right) = \underline{31^\circ}$

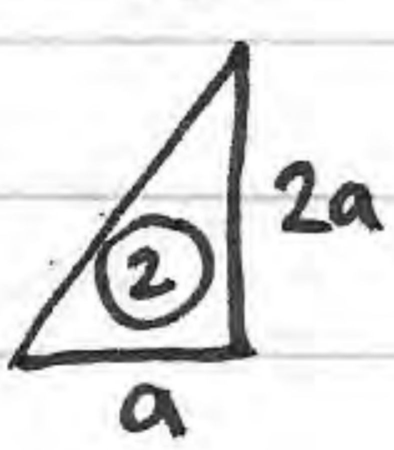
c) KE before $= \frac{1}{2}(0.2)(\sqrt{10^2})^2 = 10J$
KE after $= \frac{1}{2}(0.2)(\sqrt{15^2+15^2})^2 = 45J.$

$\therefore$ KE gain $= \underline{35J}.$

3)

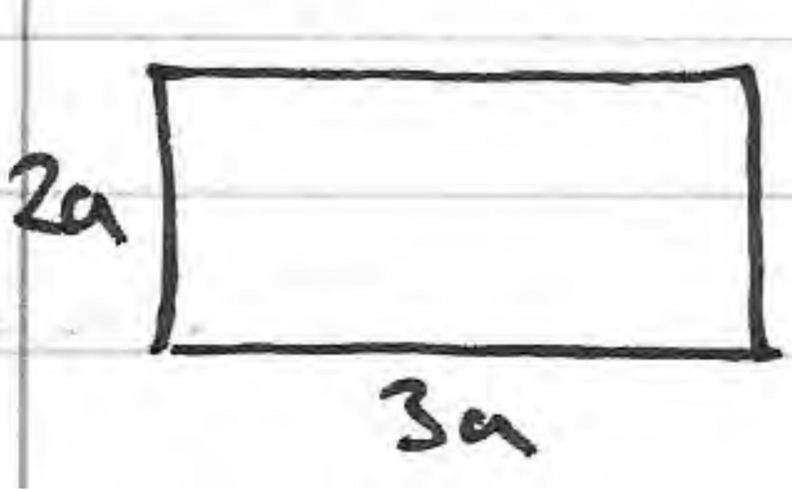


$M = \frac{(3a+2a)2a}{2}k = 5a^2k \qquad g_1(x_1, y_1)$

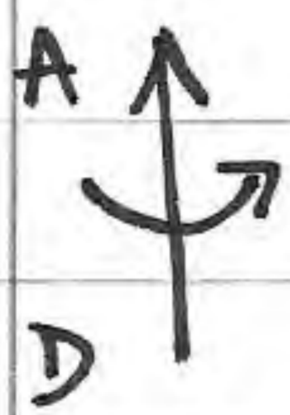


$M = \frac{a \times 2a}{2}k = a^2k \qquad g_2\left(\frac{2a+3a+3a}{3}, y_2\right)$

$g_2\left(\frac{8}{3}a, y_2\right)$



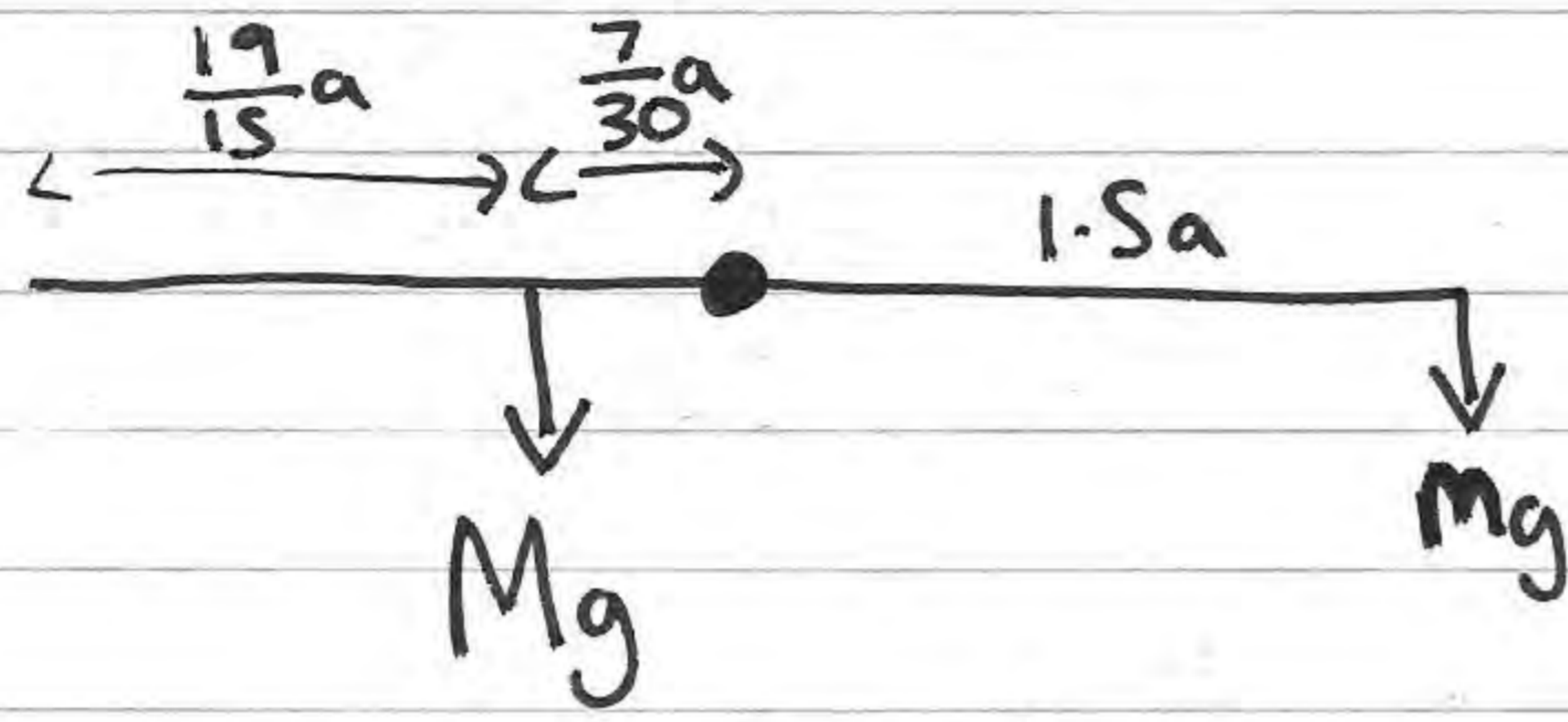
$M = 6a^2k \qquad G(1.5a, a)$



$$5\cancel{a^2}\cancel{kg}\times x_1 + \cancel{a^2}\cancel{kg}\times \frac{8}{3}a = 6\cancel{a^2}\cancel{kg}\times 1.5a$$

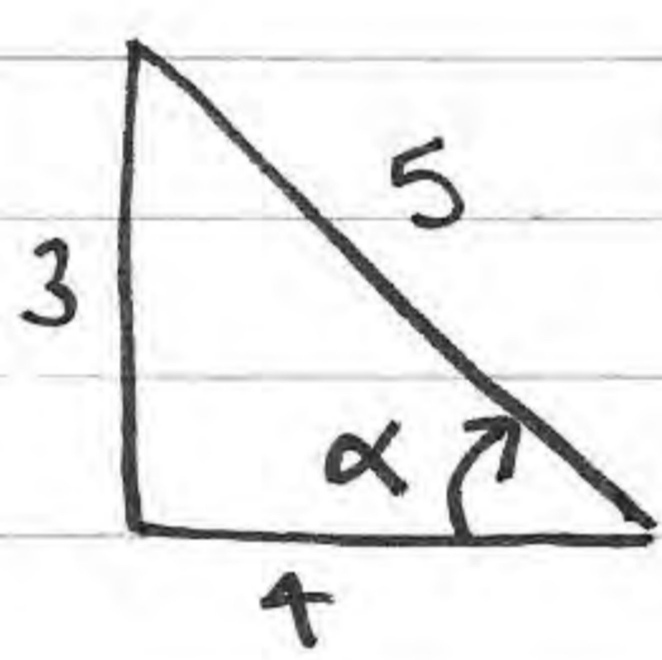
$$\Rightarrow 5x_1 + \frac{8}{3}a = 9a \Rightarrow x_1 = \frac{19}{15}a$$

b)



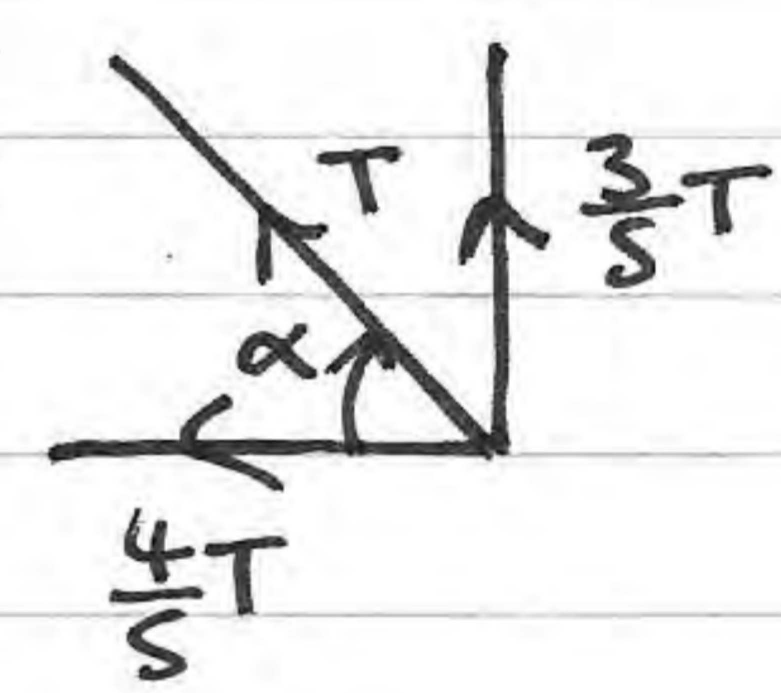
$$M\cancel{g}\times\frac{7}{30}\cancel{a} = m\cancel{g}\times\frac{3}{2}\cancel{a} \Rightarrow m = \frac{7}{45}M$$

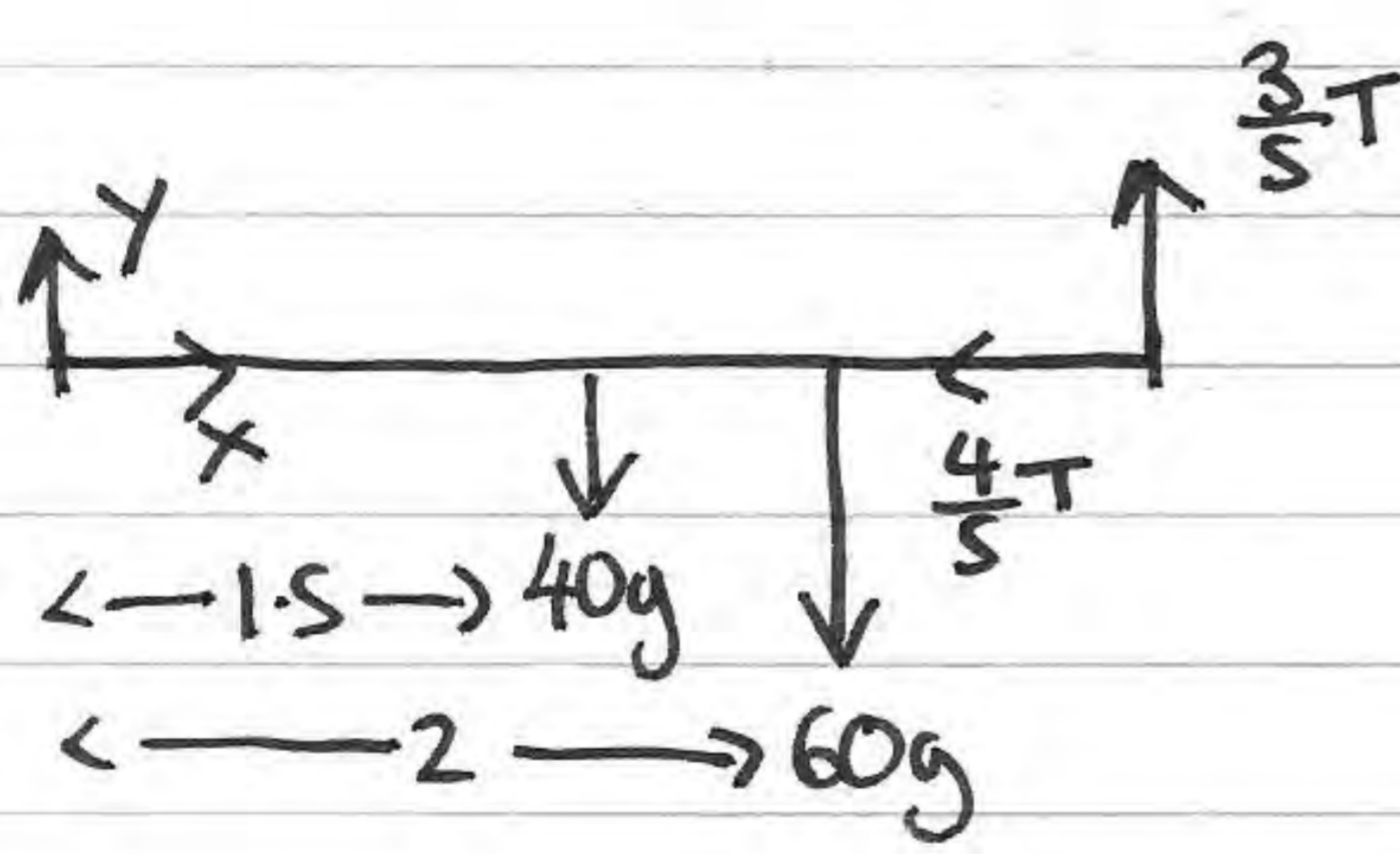
4)



$\tan\alpha = \frac{3}{4} \Rightarrow \sin\alpha = \frac{3}{5}$

$\cos\alpha = \frac{4}{5}$





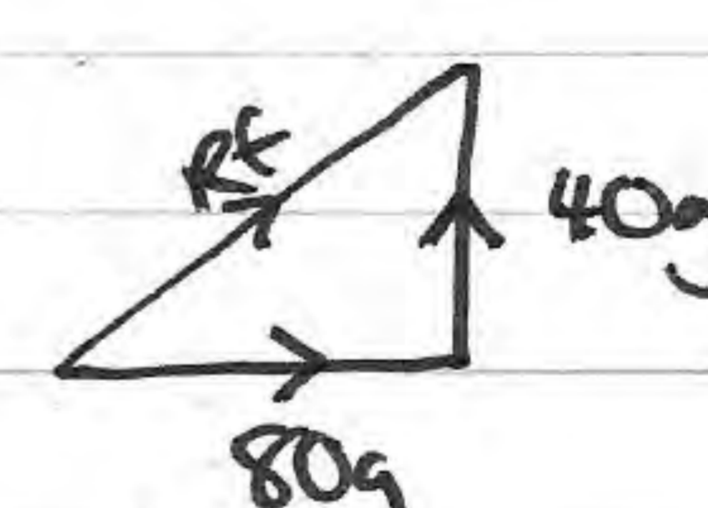
$A\circlearrowright$ $40g\times 1.5 + 60g\times 2 = \frac{3}{5}T\times 3$

$180g = \frac{9}{5}T$

$T = \underline{100g}\,N\ (980N)$ #

b) $\overrightarrow{Rf} = 0 \Rightarrow X = \frac{4}{5}T = 80g$

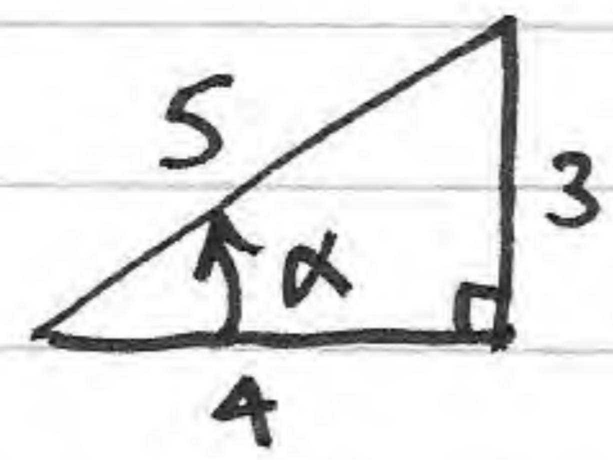
$Rf\uparrow = 0 \Rightarrow Y = 100g - \frac{3}{5}(100g) = 40g$



$Rf = \sqrt{(80g)^2 + (40g)^2} = \underline{40\sqrt{5}g}\ N$

c) Same tension, no additional weight added to system.

5)



$\tan\alpha = \frac{3}{4} \quad \cos\alpha = \frac{4}{5} \quad \sin\alpha = \frac{3}{5}$

$\overrightarrow{H}$ $\quad vel = \frac{4}{5}u \qquad s = 8 \qquad 8 = \frac{4}{5}ut \Rightarrow 40 = 4ut$

$\Rightarrow 10 = ut.$

$V\uparrow$ $\quad u = \frac{3}{5}u \qquad s = ut + \frac{1}{2}at^2$

$a = -9.8 \qquad -4 = \frac{3}{5}ut - 4.9t^2$

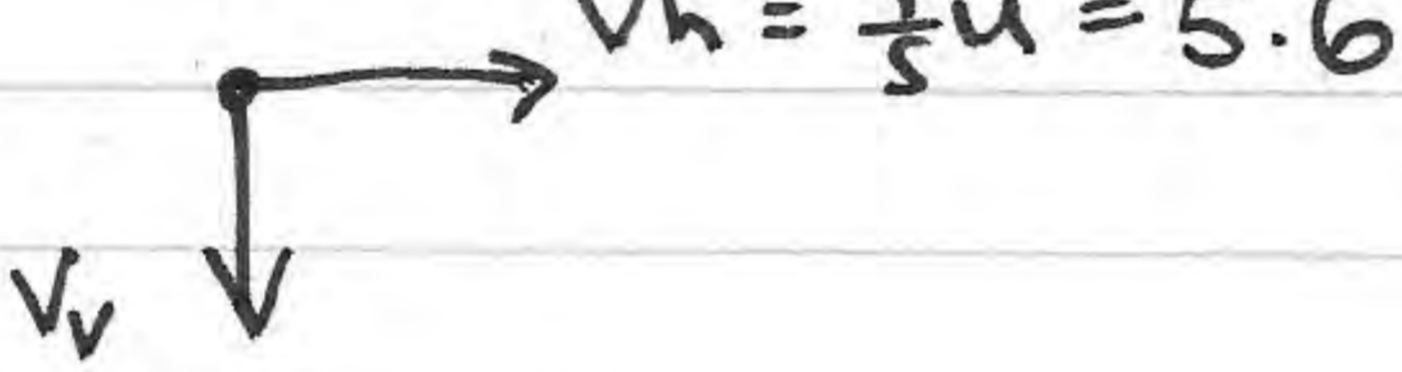
$s = -4$

$-4 = \frac{3}{5}(10) - 4.9t^2 \Rightarrow 4.9t^2 = 10$

$\Rightarrow t = \sqrt{\frac{10}{4.9}}$

$u = \frac{10}{t} = \frac{10}{\sqrt{\frac{10}{4.9}}} = \underline{7\,ms^{-1}}$

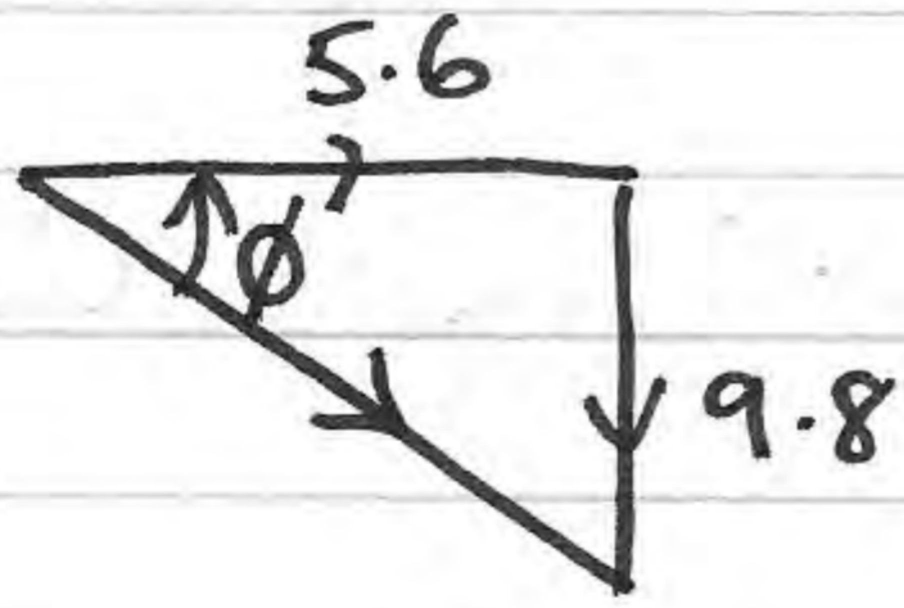
c)



$V_v\uparrow = u + at$

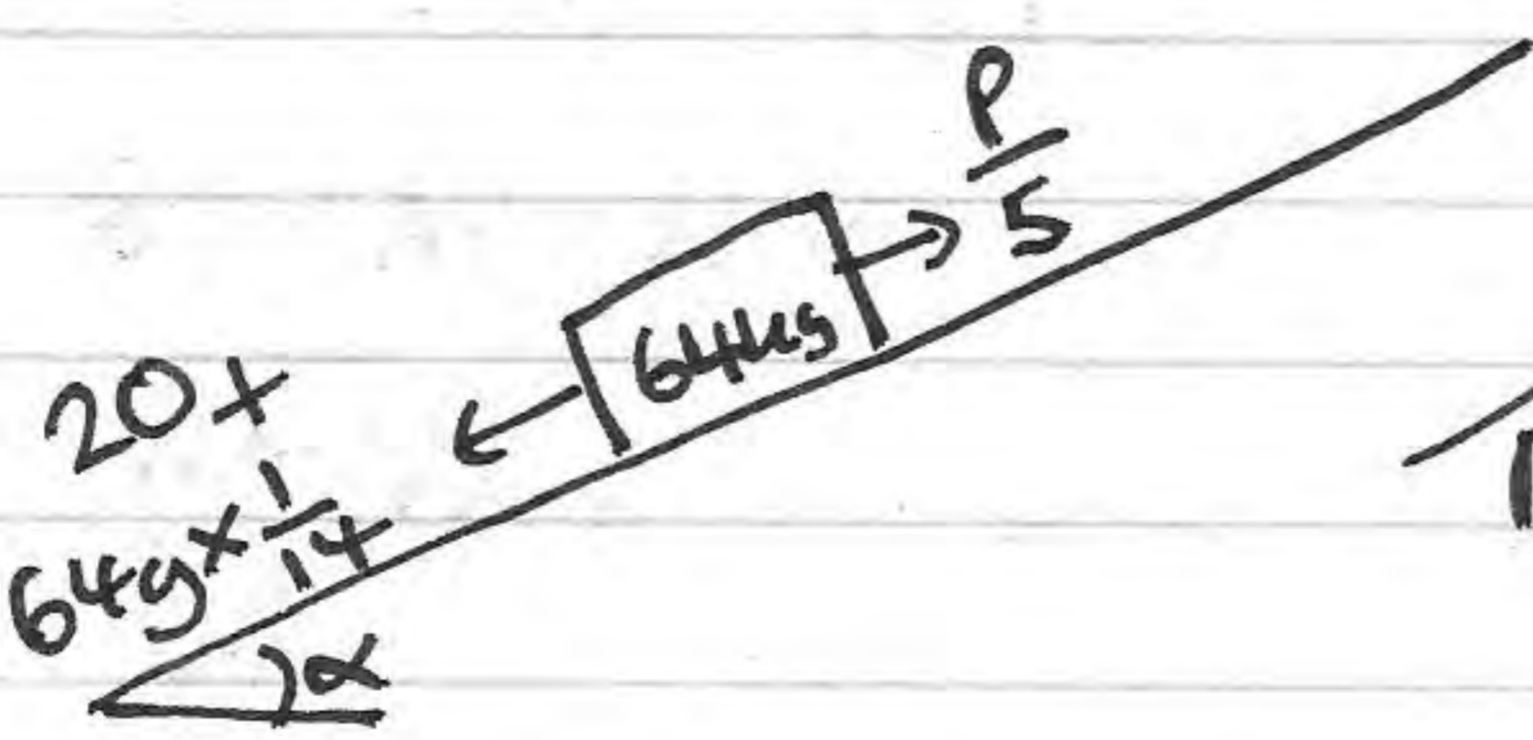
$V_v = \frac{3}{5}(7) - 9.8\left(\sqrt{\frac{10}{4.9}}\right) = -9.8$

$\Rightarrow V_v\downarrow = 9.8$



$\tan\phi = \frac{9.8}{5.6} = \underline{\underline{\frac{7}{4}}}$

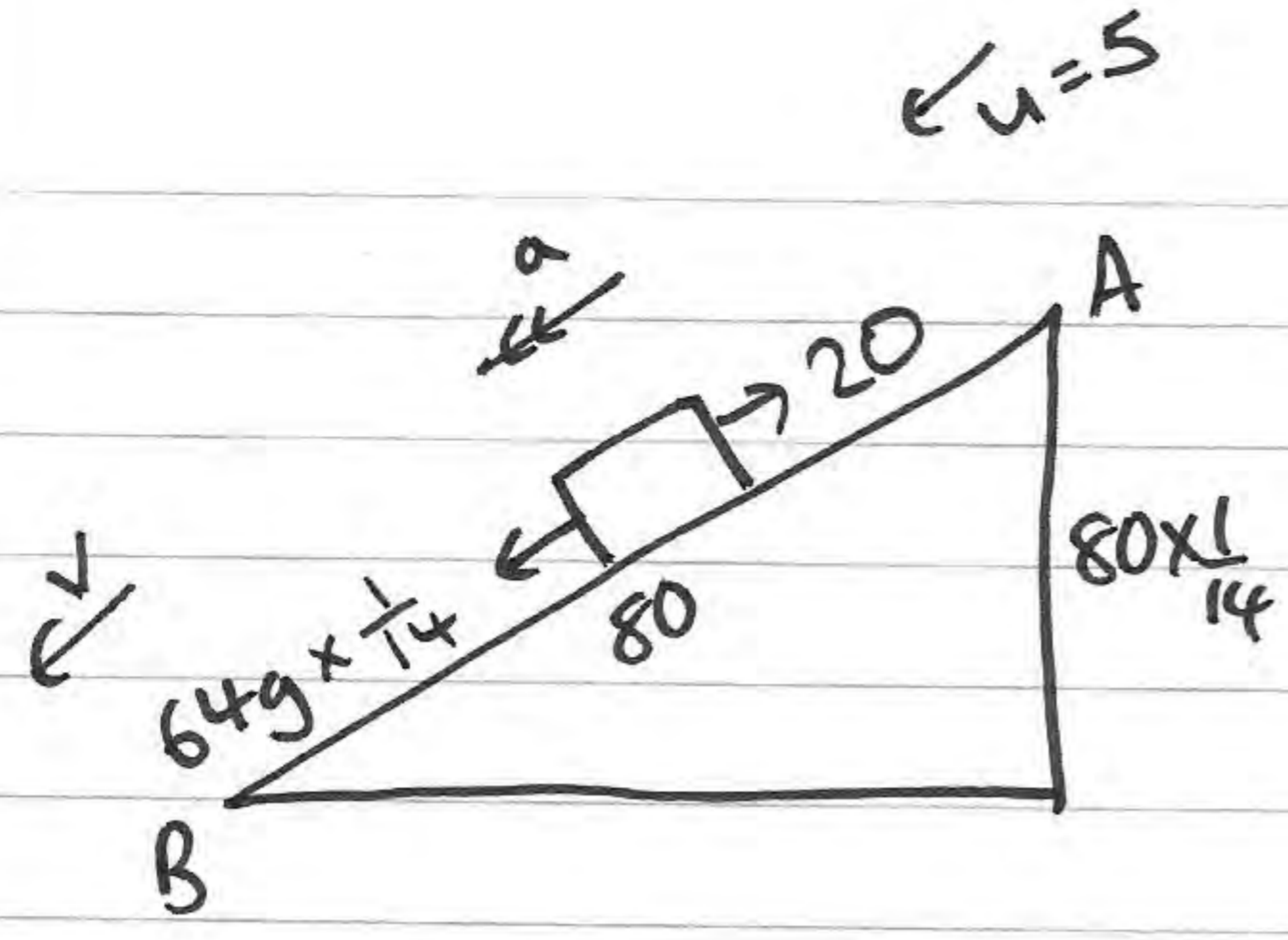
6)



$\sin\alpha = \frac{1}{14}$

$\nearrow R_f = 0 \Rightarrow \frac{P}{5} = 20 + \frac{64g}{14}$

$\Rightarrow \underline{P = 324w}$



$KE_A + PE_A - \text{Wd against Res} = KE_B + PE_B^{(=0)}$

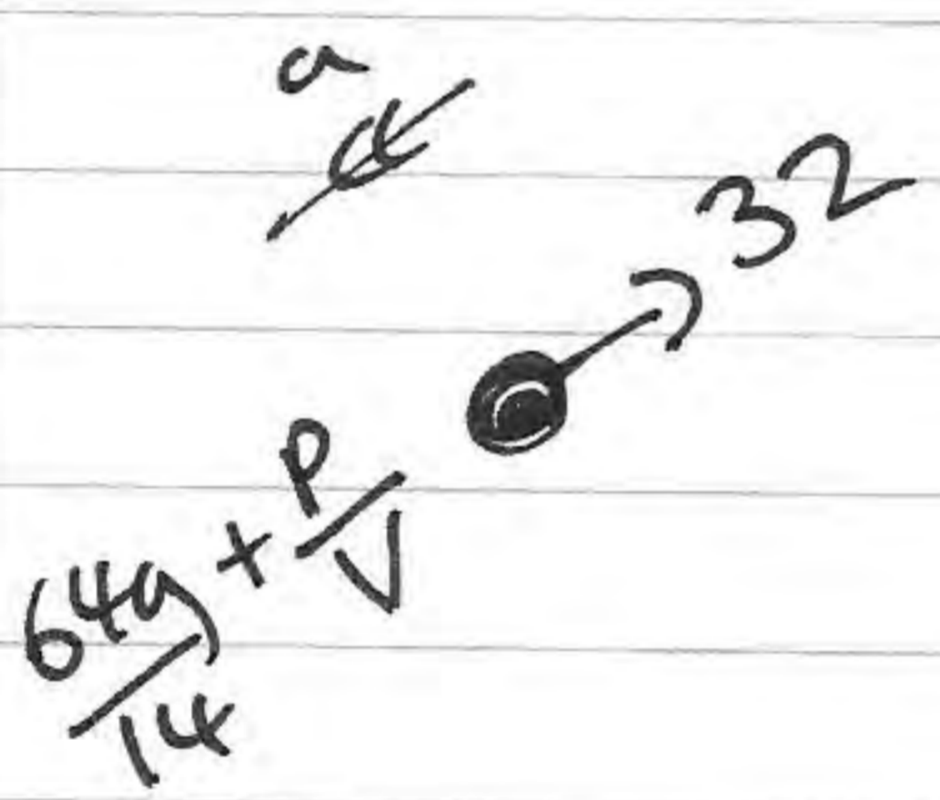
$\frac{1}{2}(64)(5)^2 + 64g \times 80 \times \frac{1}{14} - 20 \times 80 = \frac{1}{2}(64)v$

$\Rightarrow V = \sqrt{87} = \underline{9.3\,ms^{-1}}$ (2sf)

c) $R \propto V \Rightarrow R = kV$ $\qquad 20 = k \times 5 \Rightarrow k = 4$

$Res = 4 \times 8 \Rightarrow \underline{R = 32N}$

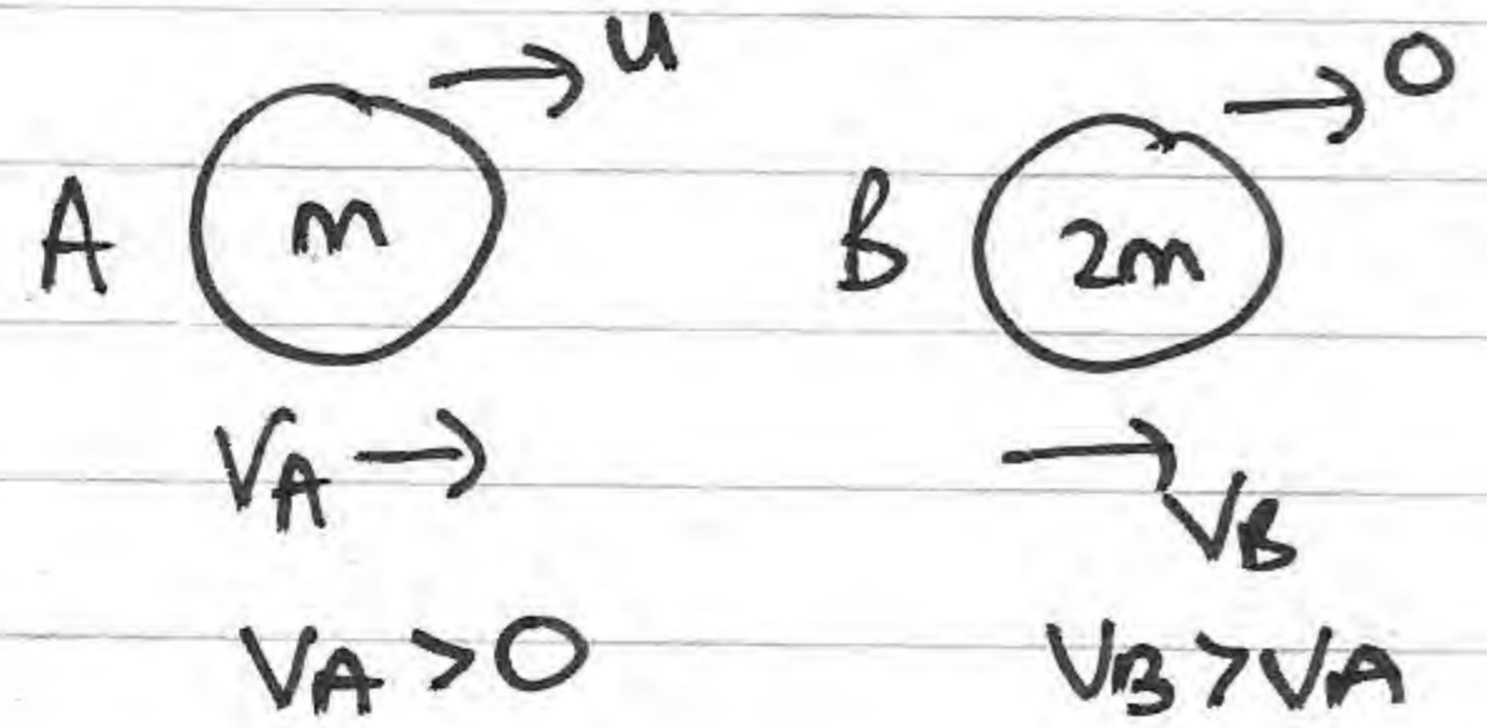
d)



$R + \swarrow = ma \Rightarrow \frac{64g}{14} + \frac{200}{8} - 32 = 64a$

$\Rightarrow a = \underline{0.59\,ms^{-2}}$ (2sf)

7)



$CLM \Rightarrow \cancel{m}u = \cancel{m}V_A + 2\cancel{m}V_B$

$\Rightarrow u = V_A + 2V_B$

$e = \frac{Sep}{app} = \frac{V_B - V_A}{u}$

$eu = V_B - V_A \Rightarrow V_B = eu + V_A$

$\therefore u = V_A + 2eu + 2V_A \Rightarrow 3V_A = u - 2eu \Rightarrow \underline{V_A = \frac{1}{3}u(1-2e)}$

$V_B = eu + \frac{1}{3}u(1-2e) = eu + \frac{1}{3}u - \frac{2}{3}ue = \frac{1}{3}eu + \frac{1}{3}u$

$V_B = \frac{1}{3}u(1+e)$

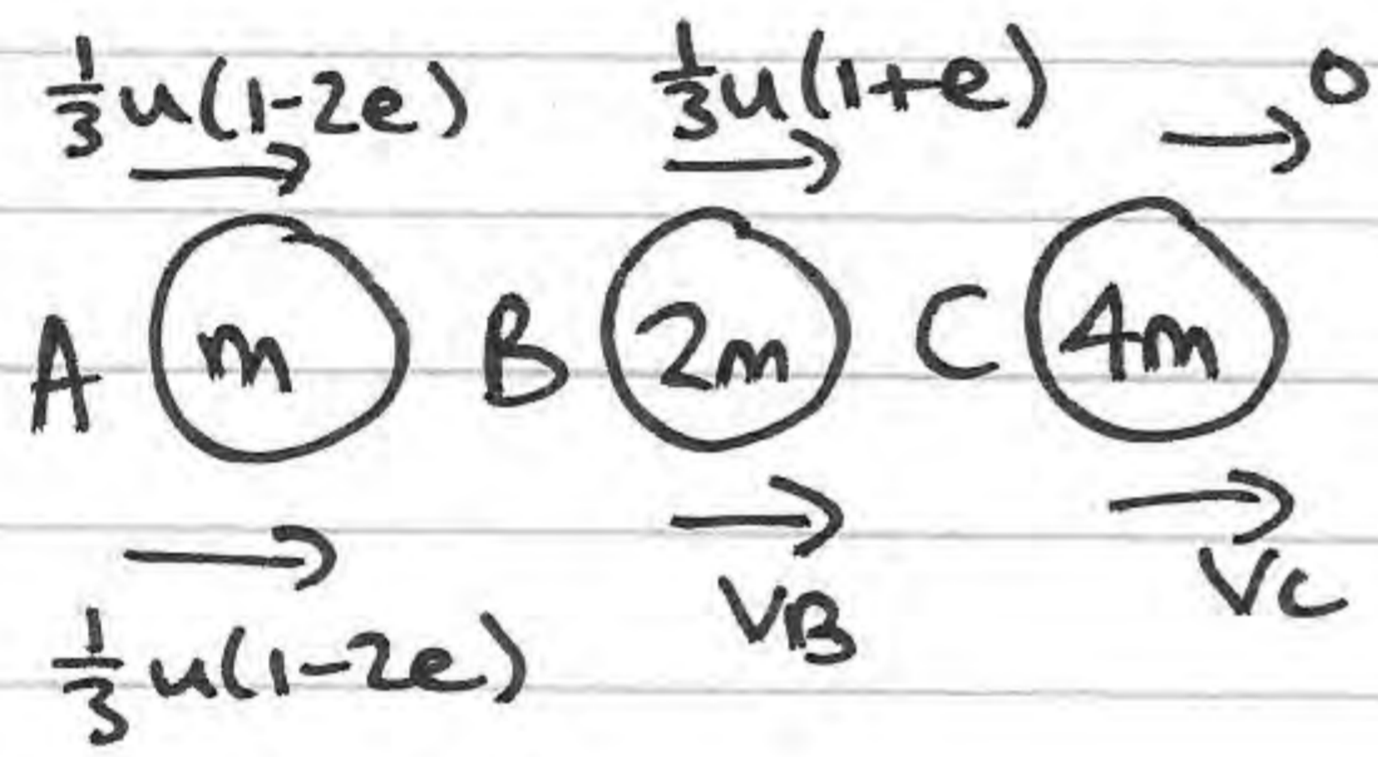
b) $V_B > V_A \Rightarrow \cancel{\tfrac{1}{3}}u(1+e) > \cancel{\tfrac{1}{3}u}(1-2e)$

$\Rightarrow 1+e > 1-2e \Rightarrow 3e > 0 \Rightarrow e > 0$

$V_A > 0 \Rightarrow \frac{1}{3}u(1-2e) > 0 \Rightarrow 2e < 1 \Rightarrow e < \frac{1}{2}$

$\therefore \quad 0 < e < \frac{1}{2}$

c)



$e = \frac{sep}{app} = \frac{V_C - V_B}{\frac{1}{3}u(1+e)}$

$\Rightarrow \frac{1}{3}eu(1+e) = V_C - V_B$

$V_C = \frac{1}{3}eu(1+e) + V_B$

CLM $\Rightarrow 2\cancel{m}\left(\frac{1}{3}u(1+e)\right) = 2\cancel{m}V_B + 4\cancel{m}V_C$

$\Rightarrow \frac{2}{3}u(1+e) = 2V_B + \frac{4}{3}eu(1+e) + 4V_B$

$\Rightarrow \frac{2}{3}u(1+e) - \frac{4}{3}eu(1+e) = 6V_B$

$\Rightarrow \frac{2}{3}u(1+e)(1-2e) = 6V_B \Rightarrow V_B = \frac{1}{9}u(1+e)(1-2e)$

$e < \frac{1}{2} \Rightarrow V_B > 0 \qquad e > 0 \Rightarrow V_B < \frac{1}{9}u$

$V_B = \frac{1}{9}u(1+e)(1-2e) = \frac{1}{3}(1+e)\left[\frac{1}{3}u(1-2e)\right]$

$V_B = \frac{1}{3}(1+e)V_A \qquad 0 < e < \frac{1}{2} \qquad \frac{1}{3}V_A < V_B < \frac{1}{2}V_A$

$\therefore$ since $V_B > 0$ and $V_B < V_A$ they must collide again